

$$\begin{aligned}
 1) & (\lambda x. \overset{\uparrow\uparrow}{xx}y)(\lambda x. \lambda y. \overset{\uparrow}{xy}y) \rightarrow (\lambda x. \lambda y. \overset{\uparrow}{xy}y)(\lambda x. \lambda y. \overset{\uparrow}{xy}y) y \\
 & (\cancel{\lambda y. (\lambda x. \lambda y. xy) yy}) y \rightarrow (\lambda x. \lambda y. xy) yy \equiv_{\alpha} (\lambda x. \lambda z. xzz) yy \\
 & \rightarrow (\lambda z. yzz) y \rightarrow yyy \quad \checkmark
 \end{aligned}$$

$$\begin{aligned}
 2) & (\lambda y. y(\lambda z. xz)) \{x := \lambda z. y\} \rightarrow (\lambda y. y(\lambda z. (\lambda z. y)z)) \quad \checkmark \\
 & (x(\lambda y. \lambda x. x)) \{x := (yz)\} \rightarrow ((yz)(\lambda y. \lambda x. x)) \quad \checkmark \\
 & (x(\lambda y. \lambda x. zx)) \{x := \lambda y. y\} \{z := x\} \rightarrow (\lambda y. y(\lambda y. \lambda x. zx)) \equiv_{\alpha} \\
 & \quad (\lambda y. y(\lambda y. \lambda k. zk)) \rightarrow \\
 & \quad (\lambda y. y(\lambda y. \lambda k. x.k)) \quad \checkmark
 \end{aligned}$$

$$\begin{aligned}
 3) & (\cancel{\lambda x. ((\lambda x. \lambda y. yy)(\lambda x. (\lambda y. yy)(\lambda x. xx)))})x \rightarrow \\
 & ((\lambda x. \lambda y. yy)(\lambda x. (\lambda y. yy)(\lambda x. xx))) \rightarrow \\
 & ((\lambda x. \lambda y. yy)(\lambda x. (\lambda x. xx)(\lambda x. xx))) \rightarrow \text{Infinita (call-by-value)} \quad \checkmark \\
 & (\cancel{\lambda x. ((\lambda x. \lambda y. yy)(\lambda x. (\lambda y. yy)(\lambda x. xx)))})x \rightarrow \\
 & ((\lambda x. \lambda y. yy)(\lambda x. (\lambda y. yy)(\lambda x. xx))) \rightarrow \lambda y. yy \quad (\text{call-by-name}) \quad \checkmark \\
 & \quad \text{NON CI SONO OCCORRENZE DI } x
 \end{aligned}$$

$$\begin{aligned}
 4) & (\lambda x. \lambda y. \overset{\uparrow}{yx})(\lambda x. \lambda y. x(\overset{\uparrow}{yy}))(\lambda x. xz(\lambda y. yy)) \rightarrow \\
 & (\lambda y. y(\lambda x. \lambda y. x(\overset{\uparrow}{yy}))) (\lambda x. xz(\lambda y. yy)) \rightarrow
 \end{aligned}$$

$$(\lambda x. x z (\lambda y. yy)) (\lambda x. \lambda y. x (yy)) \rightarrow (\lambda x. \lambda y. x (yy)) z (\lambda y. yy) \rightarrow$$

$$(\lambda y. z (yy)) (\lambda y. yy) \rightarrow z (\lambda y. yy) (\lambda y. yy) \rightarrow \text{Infinita} \checkmark$$

$$5) (\lambda x. x) (\lambda x. \lambda y. x) (xy) \equiv_{\alpha} (\lambda x. x) (\lambda x. \lambda z. x) (xy) \rightarrow$$

$$(\lambda x. x) (\lambda z. xy) \rightarrow \lambda z. xy \checkmark$$

$$6) (\lambda x. \lambda y. ((\lambda y. yx) (\lambda x. xy))) (xy) \rightarrow \lambda y. ((\lambda y. yx) (\lambda x. xy)) \rightarrow$$

$$\lambda y. ((\lambda x. xy) x) \rightarrow \lambda y. xy$$

$$7) (\lambda x. (\lambda y. xy)) y \equiv_{\alpha} (\lambda x. (\lambda z. xz)) y \rightarrow \lambda z. yz$$

$$8) (\lambda x. \lambda y. ((\lambda z. z) (xy))) K \rightarrow \lambda y. ((\lambda z. z) (ky))$$

$$\lambda y. ky$$

Esempi d'esame

$$1) (\lambda x. xxy) (\lambda x. \lambda y. xyy) \rightarrow (\lambda x. \lambda y. xyy) (\lambda x. \lambda y. xyy) y$$

$$\rightarrow (\lambda y. (\lambda x. \lambda y. xyy) yy) y \rightarrow (\lambda x. \lambda y. xyy) yy \equiv_{\alpha} (\lambda x. \lambda z. xzz) yy$$

$$\rightarrow (\lambda z. yzz) y \rightarrow yyy$$

$$2) (\lambda y. y (\lambda z. xz)) \{x := \lambda z. y\} \rightarrow (\lambda y. y (\lambda z. (\lambda z. y) z))$$

$$(x (\lambda y. \lambda x. x)) \{x := (yz)\} \rightarrow ((yz) (\lambda y. \lambda x. x))$$

$$\begin{aligned}
 (x(\lambda y. \lambda x. zx)) \{x := \lambda y. y\} \{z := x\} &\rightarrow (\lambda y. y(\lambda y. \lambda x. zx)) \equiv_\alpha \\
 &(\lambda y. y(\lambda y. \lambda k. zk)) \rightarrow \\
 &(\lambda y. y(\lambda y. \lambda k. xk))
 \end{aligned}$$

$$3) (\lambda x. ((\lambda x. \lambda y. yy)(\lambda x. (\lambda y. yy)(\lambda x. xx))))x \rightarrow$$

$$\underbrace{(\lambda x. \lambda y. yy)}_{\text{Non ci sono occorrenze di } x} (\lambda x. (\lambda y. yy)(\lambda x. xx)) \rightarrow \lambda y. yy \quad \text{call-by-name}$$

$$(\lambda x. ((\lambda x. \lambda y. yy)(\lambda x. (\lambda y. yy)(\lambda x. xx))))x \rightarrow$$

$$(\lambda x. \lambda y. yy)(\lambda x. (\lambda y. yy)(\lambda x. xx)) \rightarrow$$

$$(\lambda x. \lambda y. yy)(\lambda x. (\lambda x. xx)(\lambda x. xx)) \rightarrow \text{Infinita call-by-value}$$

4)

$$(\lambda x. \lambda y. yx) (\lambda x. \lambda y. x(yy)) (\lambda x. xz(\lambda y. yy)) \rightarrow$$

$$(\lambda y. y(\lambda x. \lambda y. x(yy))) (\lambda x. xz(\lambda y. yy)) \rightarrow$$

$$(\lambda x. xz(\lambda y. yy)) (\lambda x. \lambda y. x(yy)) \rightarrow$$

$$(\lambda x. \lambda y. x(yy)) z (\lambda y. yy) \rightarrow (\lambda y. z(yy)) (\lambda y. yy) \rightarrow$$

$$z(\lambda y. yy)(\lambda y. yy) \longrightarrow \text{Infinita}$$

$$5) (\lambda x. x)((\lambda x. \lambda y. x)(xy)) \longrightarrow (\lambda x. \lambda y. x)(xy) =_\alpha$$

$$(\lambda x. \lambda k. x)(xy) \longrightarrow \lambda k. xy$$

$$6) (\lambda x. \lambda y. ((\lambda y. yx)(\lambda x. xy)))(xy) \longrightarrow$$

$$(\lambda y. ((\lambda y. yx)(\lambda x. xy)))(xy) \longrightarrow \lambda y. ((\lambda x. xy)x) \longrightarrow$$

$$\lambda y. xy$$

$$\lambda z. (((\lambda x. \lambda x. yx)x)(v \lambda z. \lambda w. v)) \longrightarrow$$

$$\lambda z. (((\lambda x. \lambda z. yz)x)(v \lambda z. \lambda w. v)) \longrightarrow$$

$$\lambda z. ((\lambda z. yz)(v \lambda z. \lambda w. v)) \longrightarrow$$

$$\lambda z. (y(v \lambda z. \lambda w. v)) \longrightarrow \text{BOH}$$

$$\lambda y. y(\lambda z. xz) \{x := \lambda z. y\} \longrightarrow$$

$$\lambda y. y(\lambda k. xk) \longrightarrow \lambda y. y(\lambda k. (\lambda z. y)k)$$

$$x(\lambda yx. x) \{x := (yz)\} \longrightarrow (yz)(\lambda yx. x)$$

$$\underbrace{(\lambda x. y y)}_{\uparrow} (\underbrace{((\lambda z. z) x)}) \longrightarrow y y$$

$$(\lambda x x x x. x x) (\lambda x. x x) (\lambda x. x) y ((\lambda x. x) x) \longrightarrow$$

$$\underbrace{(\lambda x. ((\lambda y. z) x))}_{\uparrow} (\underbrace{((\lambda x. x x) (\lambda x. x x))}) \longrightarrow \text{Infinita Call by value}$$

$$(\lambda y. z) x \longrightarrow z \quad \text{Call by name}$$

$$(\lambda w. (x w)) (\lambda y. (y x)) \quad \{ x := \lambda x. (w (x y)) \}$$

$$(\lambda K. (x K)) (\lambda z. (z x)) \longrightarrow$$

$$(\lambda K. (\lambda x. (w (x y))) \underbrace{K}_{\uparrow}) (\lambda z. z (\lambda x. (w (x y)))) \longrightarrow$$

$$(\lambda x. (w (x y))) (\lambda z. z (\lambda x. (w (x y)))) \longrightarrow$$

$$w (\underbrace{(\lambda z. z (\lambda x. (w (x y))))}_{\uparrow} y) \longrightarrow$$

$$w (\underbrace{(\lambda x. (w (x y)))}_{\uparrow} y) \longrightarrow$$

$$w (w (y y))$$

$$(\lambda x. \underbrace{xx}_\uparrow y)(\lambda x y. \underbrace{xy}_\downarrow y) \rightarrow$$

$$(\lambda x y. \underbrace{xy}_\downarrow y)(\lambda x y. \underbrace{xy}_\downarrow y) \underbrace{y}_\downarrow \rightarrow$$

$$(\lambda x y. \underbrace{xy}_\downarrow y) \underbrace{yy}_\downarrow \rightarrow yyy$$

$$(\lambda xxx. (\underbrace{(\lambda xxx. x)(xx)(\lambda x. x)}_\uparrow)) \underbrace{x}_{\downarrow} (xx) \rightarrow$$

$$(\lambda xxx. (\underbrace{(\lambda xxx. x)(xx)(\lambda x. x)}_\uparrow)) (\underbrace{xx}_\downarrow) \rightarrow$$

$$\lambda x. (\underbrace{(\lambda xxx. x)}_\uparrow (\underbrace{xx}_\downarrow) (\lambda x. x)) \rightarrow$$

$$\lambda x. (\lambda x. x)(\lambda x. x) \rightarrow \lambda x. (\lambda x. x)$$

$$(\lambda x. \lambda y. y)((\lambda y. \underbrace{yy}_\uparrow z)(\lambda y. \underbrace{yz}_\downarrow)) \rightarrow$$

$$(\lambda x. \lambda y. y)((\lambda y. yz)(\lambda y. \underbrace{yz}_\downarrow) \underbrace{z}_\downarrow) \rightarrow$$

$$(\lambda x. \lambda y. y)((\lambda y. \underbrace{yz}_\uparrow) \underbrace{zzz}_\downarrow) \rightarrow$$

$$(\lambda x. \lambda y. y)(\underbrace{zzzzz}_\downarrow) \rightarrow zzzzz$$

↑
No occ.